On the modelling of the optical properties of a metasurface composed of nanofins

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Abstract:
The phase shift imparted by a dielectric nanofin metasurface can be modelled following a semi-analytical assessment of the guiding properties. The model can be seen as a correction to the typically used Pancharatnam-Berry model.

1. Introduction

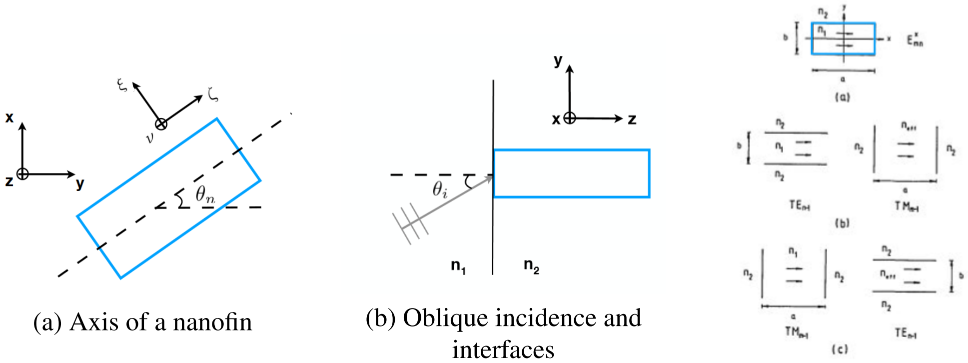
Dielectric structures made of high refractive index material has shown significant efficiency improvement, reaching close to 100% for optical wavelengths compare to metallic metasurface [1]. These particular implementations use the geometrical phase to control light properties. To perform the design of such a metasurface, two methods are classically used. The orientation of the nanostructure is approximated by the Pancharatnam-Berry phase [2] to perform an initial design; then, the effect of a rotation of a single nanostructure is computed by finite-difference time-domain (FDTD) simulations which are extrapolated to obtain the properties of the array. The design could also be directly performed by fullwave simulations of the entire lens [1]. No model exists to approximate the optical properties of dielectric metasurfaces, which could be used as a fast means to produce an initial design.

In this paper, we propose a technique involving computing the propagation properties inside the materials constituting the structure, the transmission coefficients at the interfaces between materials and a simplification using the effective medium theory [3]. This expands earlier work performed by the authors [4].

2. Model

2.1. *Geometry of a nanofin*

The considered nanofin can be seen as a dielectric rectangular waveguide with lateral subwavelength size. The spacing between the various structures is assumed to be smaller than the wavelength. The rotation angle of the nanofin θ_n is defined as the rotation angle about axis z (i.e., the propagating axis). This angle implements a change of coordinates of the incident field toward the fast and slow axes of the birefringent structure affecting the output phase of the nanofin.



(a) Axis of a nanofin

(b) Oblique incidence and interfaces

(c) Chiang decomposition [3]

Fig. 1. Geometry of a nanofin and scheme to obtain its effective index.

2.2. *Effective medium theory*

The effective medium theory enables analytic treatment of complex structures by approximating them as simple ones with equivalent optical properties. Chiang proposed a scheme to compute the effective propagation constant [3] (see Fig. 1). One needs to solve the transcendental propagation constants k_{sol} of the TM or TE modes along one dimension, obtain the effective index $n_{eff} = k_{sol}\lambda/2\pi$ and then use it for the relation along the other dimension and mode. By doing so, the birefringence of the nanofin can be accounted for.

2.3. *Transmission coefficients*

The transmission coefficients for the different polarizations at one interface are simply the Fresnel coefficients.

2.4. *Implemented phase*

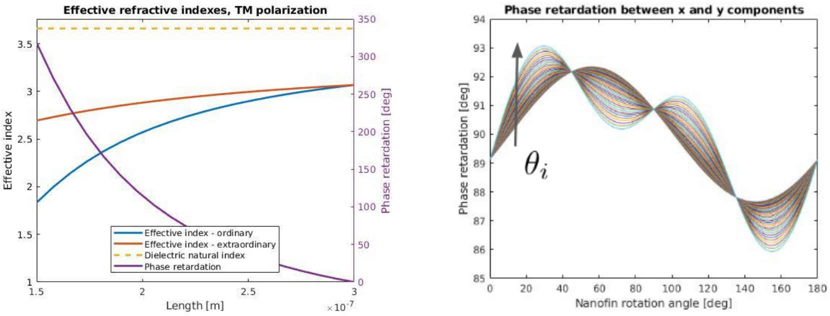
The effective index is computed for the fundamental mode only. The various phase constant $\beta_{\xi;\zeta;\nu}$ of the structure are obtained using the effective medium method. Then, using the uncorrected Snell-Descartes, one obtains the transmission angles at the first and second interfaces. The E_x field component after a nanofin is given by

$$E_x = \{\tau_\xi^{\|} \cos\theta_i \cos\theta_n - \tau_\xi^{\perp} i \sin\theta_n\} \cos\theta_n e^{i\beta_\xi H} + \{\tau_\zeta^{\|} \cos\theta_i \sin\theta_n + \tau_\zeta^{\perp} i \cos\theta_n\} \sin\theta_n e^{i\beta_\zeta H}. \tag{1}$$

To obtain the implemented phase ϕ, one need to compute the argument of this field. Note that for the idealized case, the phase implemented ϕ by the metasurface follows exactly the Pancharatnam-Berry phase, i.e. $\phi = 2\theta_n$ [2].

3. Verification of the model

The effective index of refraction and phase retardation have been studied by Khorasaninejad et al. using FDTD simulations [5]. Results of the proposed method are presented in Fig. 2(a). One can observe that the effective index of refraction of a nanofin depends upon its size. Our results are qualitatively equivalent to the ones from Khorasaninejad et al. with a small variation in phase retardation. Fig. 2(b) shows that the properties of the nanofin are differently affected for different couple (θ_i,θ_n).



(a) Effective medium theory (b) Incidence angle effect

Fig. 2. Optical properties of a single nanofin. (a) Phase variation and effective index (b) effect of incidence angle on the phase retardation of a TiO_2 nanofin of 110x300x1000 nm^3 at λ = 575 nm.

4. Conclusion

We presented a semi-analytical model to obtain the optical properties of metasurface based on space-varying dielectric nanofins. To do so, effective medium theory, transmission coefficients and the propagation inside the structure are accounted for. Results are in good agreement with the literature and can be seen as an extension of the idealized Pancharatnam-Berry phase. Our model shows incident angle variations of the phase retardance which can not be predicted by the Pancharatnam-Berry approach.

References

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